Spin 1/2 - matrix representation

Pauli matrices

Definition

In[1]:=
```
MatrixForm[σx = {{0, 1}, {1, 0}}]
```
Out[1]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[2]:=
```
MatrixForm[σy = {{0, -I}, {I, 0}}]
```
Out[2]//MatrixForm=

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

In[3]:=
```
MatrixForm[σz = {{1, 0}, {0, -1}}]
```
Out[3]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Algebra

In[4]:=
```
MatrixPower[σx, 2] // MatrixForm
```
Out[4]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[5]:=
```
MatrixPower[σy, 2] // MatrixForm
```
Out[5]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[6]:=
```
MatrixPower[σz, 2] // MatrixForm
```
Out[6]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[7]:=
```
σx.σy + σy.σx // MatrixForm
```
Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[8]:=
```
σx.σz + σz.σx // MatrixForm
```
Out[8]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[9]:=
```
σy.σz + σz.σy // MatrixForm
```
Out[9]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[10]:= `σx.σy - I * σz // MatrixForm`

Out[10]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[11]:= `σz.σx - I * σy // MatrixForm`

Out[11]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[12]:= `σy.σz - I * σx // MatrixForm`

Out[12]//MatrixForm=

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

In[13]:= `σx.σx // MatrixForm`

Out[13]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[14]:= `σy.σy // MatrixForm`

Out[14]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

In[15]:= `σz.σz // MatrixForm`

Out[15]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Eigenvalues and normalized eigenvectors

In[16]:= `Eigenvalues[σx]`

Out[16]= $\{-1, 1\}$

In[17]:= `Map[Normalize, Eigenvectors[σx]]`

Out[17]= $\left\{\left\{-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[18]:= `Eigenvalues[σy]`

Out[18]= $\{-1, 1\}$

In[19]:= `Map[Normalize, Eigenvectors[σy]]`

Out[19]= $\left\{\left\{\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}, \left\{-\frac{i}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\}\right\}$

In[20]:= `Eigenvalues[σz]`

Out[20]= $\{-1, 1\}$

In[21]:= `Map[Normalize, Eigenvectors[σz]]`

Out[21]= $\{\{0, 1\}, \{1, 0\}\}$

Matrix representation of the spin components for a spin

of magnitude $\hbar/2$

In[22]:=
```
Sx = (ℏ / 2) * σx; Sy = (ℏ / 2) * σy; Sz = (ℏ / 2) * σz;
```

These matrices verify the algebra of an angular momentum

In[23]:=
```
Sx.Sy - Sy.Sx == I ℏ Sz
```

Out[23]=
```
True
```

In[24]:=
```
Sz.Sx - Sx.Sz == I ℏ Sy
```

Out[24]=
```
True
```

In[25]:=
```
Sy.Sz - Sz.Sy == I ℏ Sx
```

Out[25]=
```
True
```

The general rule $\boldsymbol{L}_x^2+\boldsymbol{L}_y^2+\boldsymbol{L}_z^2 = j(j+1)\,\hbar^2\,\mathbf{1}$ for an angular momentum $j\hbar$ (j=1/2,1,3/2,2,...) becomes here $\boldsymbol{S}_x^2+\boldsymbol{S}_y^2+\boldsymbol{S}_z^2 = (3/4)\,\hbar^2\,\mathbf{1}$:

In[26]:=
```
MatrixPower[Sx, 2] + MatrixPower[Sy, 2] + MatrixPower[Sz, 2] ==
  3 ℏ ^ 2 / 4 IdentityMatrix[2]
```

Out[26]=
```
True
```